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Application Note:
Core Loss and Capacity

Prepared for: General

1   Summary

Have you ever wondered how much power you can get through a given core? Wonder no longer. As long as you can find manufacturer data on the part, you can probably determine a reasonable upper limit with respect to permissible temperature rise, frequency and even quality (Q) factor.

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2   Introduction

Core manufacturers often provide tools and resources describing how to choose their products based on inductance or transformer size, but thermal limitations are less often considered. With the relatively high losses of the materials, this is not a small problem. As far as choosing a particular core by thermal limit, no manufacturer provides such a resource. The intrepid engineer is left to design by trial and error, picking cores at random until one seems to fit the design equations. The resulting selection is suboptimal, often either too small and it overheats, or too large and overly expensive. Although the complete design process still can't be encapsulated in a single formula, the formulas in this paper should come in handy.

Note: MMKS units are used in this document. This is SI standard MKS units, with meters changed to milimeters; in most places, this pushes a micro somewhere else convenient, for example, frequency is in MHz, capacitance in μF, inductance in μH, flux in μWb, etc. Turns are also treated as a unit, as consistently as possible. Magnetic flux density (tesla) has the somewhat unwieldly expansion μWb mm^{-2} t^{-1}, but the units serve nicely in dimensional analysis to remember which variables are correct.

3   Analysis

The governing equations in this analysis come from basic magnetic and electrical theory. First, let's consider what we want to determine: we need the reactive power density of a core material, or the power handling of a given core. Without a consistent formula for the core losses, we default to the characteristic curves given by the manufacturer, which are usually in terms of power loss density p_c, as a function of flux density B_{max} for a range of power loss density at a selection of frequencies. So we shall select points from the graph and work from there. The next thing we need to know is how to get reactive power out of that: the best way is to use the core properties and solve for the voltage and current we would apply to an arbitrary inductor wound on that core.

Suppose we selected the T68-26A core from Micrometals. Its properties are shown in Table 1. Given the core parameters, we can also calculate A_L:
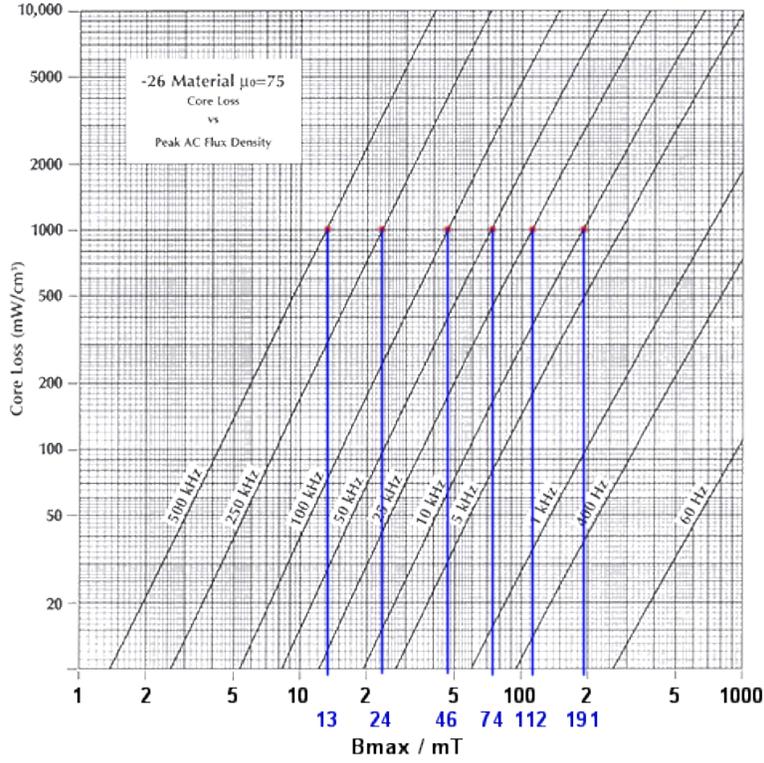
Figure 1: Core loss for Mix #26 (Micrometals, Inc.).

$$A_L = \mu_r \mu_0 \frac{A_e}{l_e} \qquad (1)$$

Which, taking $\mu_0 = 0.00126\,{}^{\mu H}/_{mm}$, gives $0.054\,{}^{\mu H}/_{t^2}$, close enough to the published value. Wound with wire, it looks like Figure 2, a commercial piece found out in the lab (the core can be identified by its yellow/white color code). With $33\,t$, evidently this is a $L = A_L N^2 = 63\,\mu H$ coil.

If we applied a voltage to this coil, a certain amount of flux density would be achieved in the core:

$$B_{max} = \frac{V_{rms}}{\sqrt{2}\,\pi F N A_e} \qquad (2)$$

Symbol	Qty.	Unit
O.D.	17.5	mm
I.D.	9.4	mm
Ht.	6.35	mm
μ_r	75	
A_L	0.058	${}^{\mu H}/_{t^2}$
l_e	42.3	mm
A_e	24.2	mm^2
v_e	1030	mm^3

Table 1: Properties of the T68-26A core. (2)

(the factor $\sqrt{2}\,\pi$ is the familiar "4.44" commonly seen) which, knowing the frequency as well, we can now find how much loss it would dissipate (due to the core alone). To find the maximum voltage at a given frequency, for a given

core, invert the equation:

$$V = \sqrt{2}\,\pi B_{max} F N A_e \tag{3}$$

Since this coil is $63\,\mu$H, if we applied a $1\,\text{V}_{rms}$ sine wave at $0.1\,\text{MHz}$, it would have a reactance of $X_L = 2\pi F L = 39.6\,\Omega$, carry a current of $I = {}^V\!/_R = 0.025\,\text{A}$, and a reactive power of $0.025\,\text{VA}$. Hopefully, the real power dissipated is not $25\,\text{mW}$, but this coordinate ($2.8\,\text{mT}$ at $100\,\text{kHz}$) doesn't appear on the graph, so we can't tell how much is dissipated. Instead of guessing how much voltage to apply, we can apply (3).

How much loss is too much? A good rule of thumb is $1000\,{}^{\text{mW}}\!/_{\text{cm3}}$, but this depends on core size, air flow and so on. A small core might tolerate much higher loss density, or a medium core with air flow, but a very large core might overheat internally, even with cooling. The T68-26A might dissipate one or two watts comfortably, maybe just one when copper losses are included in the final analysis. With a volume of nearly $1000\,\text{mm}^3$, this is a good ballpark figure to go with.

Figure 2: Choke wound on T68-26A core, 33 turns, $18\,\text{AWG}$.

At $100\,\text{kHz}$, we see $1000\,{}^{\text{mW}}\!/_{\text{cm3}}$ is reached at $46\,\text{mT}$ (Figure 1). Using (3), we find a maximum voltage of $16.3\,\text{V}$, which would be acceptable for, say, a low voltage buck or boost converter, running in continuous conduction mode (CCM). The AC current draw is $0.41\,\text{A}$, so the reactive power handled is $6.7\,\text{VA}$. The power dissipation was $1\,\text{W}$, so the quality factor Q is a paltry 6.7. Judging by the wire size, this coil should easily handle $5\,\text{A}$ or so, DC (the bias will reduce inductance a little, but not too much). This suggests not just a CCM type converter, but one with an overall power capacity of ballpark $16\text{V} \times 5\text{A} = 80\,\text{W}$, so even though the Q is crappy, it's more than made up for with the large DC component.

This is all well and good, but we still don't have a solution for picking cores yet. If we bring together all the circuit parameters (volts, amps, inductance) with all the physics (power and flux densities), we can eliminate some

variables and finish this up:

$$S = |V||I| \qquad\qquad I = \frac{V}{X_L} \qquad\qquad X_L = 2\pi F L$$

$$V = \sqrt{2}\,\pi B_{max} F N A_e \qquad\qquad L = A_L N^2 \qquad\qquad A_L = \mu_r \mu_0 \frac{A_e}{l_e}$$

Putting them all together, V, I and N drop out, so it's no longer dependent on circuit-level quantities! A_e and l_e appear together, suggesting they should be paired as $A_e l_e = v_e$. These are not necessarily the same quantities; fortunately, the product is very close to the published v_e on most cores.

$$S = \frac{\pi F B_{max}{}^2 A_e l_e}{\mu_r \mu_0} = \frac{\pi F B_{max}{}^2 v_e}{\mu_r \mu_0} \tag{4}$$

Important features of this equation:

- It depends on $B_{max}{}^2$, which makes sense (power goes as V^2, and V and B are proportional). It's also proportional to volume (check).

- Ideal Steinmetz core loss, when solved for constant power, has $F \propto B^{-2}$; since our formula for S is proportional to F and B^2, such a core works just as well at any frequency (it has constant Q). Real cores have different ratios, but if Q isn't in the right order of magnitude for your application, pick another material.

- S is inversely proportional to μ_r, which makes sense: higher permeability means more core-stuffs and less air gap, and it's the air gap that stores energy. Air also doesn't heat or saturate (at least, at experimentally known field intensities!). If you want to store a lot of energy, look for a low permeability material (or a large air gap, for a low equivalent permeability). Low permeability materials also tend to have lower core losses, leading to a higher Q and efficiency.

Usage: First, select a material. (Unfortunately, this formula doesn't help select a material; it's still up to the engineer's instinct, and the manufacturer's suggestions, as to which material to choose.) Within that material, determine the maximum flux density B_{max} for the chosen frequency of operation, based on either saturation or permissible core loss density. Knowing your application's reactive power demand, calculate the required core volume from:

$$v_e = \frac{\mu_r \mu_0 S}{\pi F B_{max}{}^2} \tag{5}$$

Find a nearby core in the manufacturer's catalog and evaluate it further. You'll probably end up needing a slightly larger core (maybe 50% more VA capacity) to account for waveform, copper losses (remember that lower permeability cores take more copper) and safety margin.

To calculate the Q, we know that $Q = S/P_c$ and $P_c = p_c v_e$, thus:

$$Q = \frac{\pi F B_{max}{}^2}{\mu_r \mu_0 p_c} \tag{6}$$

Which is now independent even of the core—purely a property of the material, frequency and core loss density, all of which can be read off the loss curves directly.

Ferrite datasheets often give a (small-signal) loss factor, or a plot of complex permeability (real μ' and imaginary μ'') versus frequency. Below the cutoff frequency (when $\mu' < \mu''$), we can easily relate Q to these parameters. In terms of permeability, $Q = \mu''/\mu'$. In terms of loss factor, $1/Q = \mu_i(\tan \delta / \mu_i)$ (why they don't just give $\tan \delta$, but put that "per μ_i" on instead, I have no idea).

4   Conclusion

Magnetic design remains a tedious process for the design engineer, but these formulas will at least come in handy. Determine the reactive power S your inductor has to handle, and if your application requires a certain amount of efficiency, setting a minimum Q as well. Plug these into Equations (5) and (6) and find how big a core you need. Browse the manufacturer's catalog, find a core a few sizes above the v_e you found, and see if it checks out. Further winding and thermal analysis is required for a complete design, which is unfortunately beyond the scope of this paper.

Although this design flow is intended specifically for powdered iron cores, the same data is available for many materials. Ferrites are somewhat different: since they're often used for transformers, complete loss data isn't always provided. Still, these formulas can be of help in designing high-ripple gapped ferrite core chokes.

5   Definitions

Quantity	Symbol	Units	Formula (if applicable)
Core Loss Density	p_c	W mm^{-3}	
Core Loss	P_c	W	$P_c = p_c v_e$
Effective Length	l_e	mm	
Effective Area	A_e	mm^2	
Effective Volume	v_e	mm^3	$v_e \approx A_e l_e$
Relative Permeability	μ_r	-	
Magnetic Constant	μ_0	μH mm^{-1}	$\mu_0 \approx 0.00126 \, \mu$H mm^{-1}
Flux Density	B_{max}	T	$(T \equiv \mu$Wb mm^{-2} t^{-1})
Turns	N	t	
Frequency	F	Hz	
Core Inductivity	A_L	μH$/$t^2	$A_L = \mu_r \mu_0 \frac{A_e}{l_e}$
Inductance	L	H	$L = A_L N^2$
Inductive Reactance	X_L	Ω	$X_L = 2\pi F L$
Apparent Power	S	VA	$S = |V||I|$

Table 2: Symbols used in magnetic design.

See Table 2 for a summary of quantities and some short formulas used above. Notes:

- For the most part, $v_e \approx A_e l_e$ is true within a few percent.

- Turns: Number of loops the winding makes around the magnetic flux path, or vice versa. For a toroid, this is the number of passes through the center. A toroid on a straight wire is one turn, because the path itself is the first loop.

- Reactance (X_L) is the imaginary component of impedance. When Ohm's law is used here ($V = IX_L$), there is an unwritten phase shift between V and I—we're only concerned about the magnitudes here. To prevent confusion, this is specified when calculating apparent power: the absolute value of V and I are taken.

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